Converting Julian Day Numbers

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Introduction

On the web page https://tondering.dk/claus/cal/julperiod.php, the following formulas are given:

To convert a date in the Gregorian or Julian calendar to a Julian Day number, JD, start by calculating these values:

$$a = \left\lfloor \frac{14 - month}{12} \right\rfloor \tag{1}$$

$$y = year + 4800 - a \tag{2}$$

$$m = month + 12a - 3 \tag{3}$$

For a date in the Gregorian calendar, calculate:

$$JD = day + \left\lfloor \frac{153m + 2}{5} \right\rfloor + 365y + \left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor - 32045 \tag{4}$$

For a date in the Julian calendar, calculate:

$$JD = day + \left\lfloor \frac{153m + 2}{5} \right\rfloor + 365y + \left\lfloor \frac{y}{4} \right\rfloor - 32083 \tag{5}$$

The value JD in equations 4 and 5 is the Julian Day that starts at noon TT[1] on the specified date.

The algorithm works fine for AD dates. If you want to use it for BC dates, you must first convert the BC year to a negative year (e.g., 10 BC = -9). The algorithm works correctly for all dates after 4800 BC, i.e. at least for all positive values of the Julian Day.

To convert the other way (i.e., to convert a Julian Day, JD, to a day, month, and year) these formulas can be used:

[1] Terrestrial Time.

For the Gregorian calendar, calculate:

$$a = JD + 32044 \tag{6}$$

$$b = \left\lfloor \frac{4a+3}{146097} \right\rfloor \tag{7}$$

$$c = a - \left\lfloor \frac{146097b}{4} \right\rfloor \tag{8}$$

For the Julian calendar, calculate:

$$b = 0 \tag{9}$$

$$c = JD + 32082 \tag{10}$$

Then for both calendars, calculate:

$$d = \left\lfloor \frac{4c+3}{1461} \right\rfloor \tag{11}$$

$$e = c - \left\lfloor \frac{1461d}{4} \right\rfloor \tag{12}$$

$$m = \left\lfloor \frac{5e+2}{153} \right\rfloor \tag{13}$$

$$day = e - \left\lfloor \frac{153m+2}{5} \right\rfloor + 1 \tag{14}$$

$$month = m + 3 - 12 \times \left\lfloor \frac{m}{10} \right\rfloor \tag{15}$$

$$year = 100b + d - 4800 + \left\lfloor \frac{m}{10} \right\rfloor \tag{16}$$

In the following sections I will try to explain the logic behind these formulas.

Converting To A Julian Day Number

The formulas are constructed to work based on a year that starts on 1 March with an era that started 4800 years before the Christian era.

Equations 1 to 3 make the necessary adjustments. In Equation 1, a is 1 in January and February, 0 in March through December. This value is used in Equation 2 to adjust the year, subtracting 1 from the year in January and February. Additionally, 4800 is added to the year to adjust the era and thus ensure that we will be working with positive year numbers[2].

[2] In theory, there is no reason why we could not use negative years, but sticking to positive values may be advantageous in some programming languages that do not handle integer division sensibly.

In Equation 3, m becomes 0 in March, 1 in April, ..., 9 in December, 10 in January, and 11 in February.

In other words, y and m are the year and month in an adjusted calendar that starts in March and whose year is offset by 4800.

Moving the start of the year to 1 March has two advantages: First, it moves the leap day to the end of the year; second, it gives us a repeated pattern of month lengths:

Mar, Apr, May, Jun, Jul	31, 30, 31, 30, 31
Aug, Sep, Oct, Nov, Dec	31, 30, 31, 30, 31
Jan, Feb	31, X

Each 5-month period follows the pattern 31, 30, 31, 30, 31 — a total of 153 days.

In Equations 4, 5, and 14 we encounter the term $\left\lfloor \frac{153m+2}{5} \right\rfloor$. For values of m in the range 0...11 this value grows exactly following the 31-30-pattern just described. This means that $\left\lfloor \frac{153m+2}{5} \right\rfloor$ tells us how many days there are from 1 March to the start of month m. The inverse of this formula, $\left\lfloor \frac{5e+2}{153} \right\rfloor$, where e is a day count, is used in Equation 13.

In Equation 4 the Julian Day is calculated as the sum of these five values:

- day, which is the day of the month.
- $\left\lfloor \frac{153m+2}{5} \right\rfloor$, which tells us how many days there are from the start of the current year to the start of month m.
- $365y$, which is the number of days in y years (not counting leap years).
- $\left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor$, which adds 1 for each leap year. (Years divisible by 4 are leap years, but years divisible by 100 are not, but years divisible by 400 are leap years.)
- -32045, which adjusts the value so that JD=0 falls on 1 January 4713 BC (Julian) as it should.

In Equation 5 the Julian Day is calculated in a similar manner with these two differences:

- $\left\lfloor \frac{y}{4} \right\rfloor$ adds 1 for each leap year. (Years divisible by 4 are leap years.)
- -32083 adjusts the value so that JD=0 falls on 1 January 4713 BC (Julian).

Converting From A Julian Day Number

For the Gregorian calendar, Equations 6-8 calculate three values, a, b, and c which can be briefly described thus:

a is the start of a modified Julian period.
b is the number of centuries since the start of the modified Julian period.
c is the number of days since the start of the current century.

In the Gregorian calendar 400 years contain 97 leap years. Equation 6 adjusts the start of the Julian period so that it starts with three “short” centuries with 24 leap years each (each having 36524 days) and one “long” century with 25 leap years (having 36525 days).

Equation 7 calculates the number of centuries that have passed since the start of our modified Julian period. The number of days in 400 Gregorian years is 146097, Equation 7 increases by 1 every century in the pattern: Three “short” centuries, one “long” century.

Equation 8 calculates the number of days that have passed since the start of the last century.

For the Julian calendar we use Equations 9 and 10 to calculate similar but simpler values. We set b to zero, and we simply let c be the number of days since the start of a modified Julian period.

Then for both calendars we perform the following calculations:

In Equation 11 we calculate the number of years, d, that are contained in c days. Four years contain 1461 days[3], and the formula in Equation 11 counts years of length 365, 365, 365, 366 days.

Equation 12 calculate the number of days, e, that have passed since the start of the current year.

Equation 13 and 14 converts e to months and days, using a month pattern of 31-30-31-30-31 as described previously.

Equation 15 changes the start of the month from March to January by adding 3, and subtracting 12 if $m \geq 10$.

Finally, Equation 16 calculates the year. In the Julian calendar b is 0 and d is the number of years since the start of the modified Julian period. This means that for a date in the Julian calendar, Equation 16 simplifies to

$$year = d - 4800 + \left\lfloor \frac{m}{10} \right\rfloor \tag{17}$$

Here, we take the year count, d, adjust the era by subtracting 4800, and add 1 if the current month is January or February.

For the Gregorian calendar Equation 16 calculates 100 times the number of centuries (b) plus the number of years in the current century (d). Finally, we adjust the era by subtracting 4800, and adding 1 if the current month is January or February.

[3] This is always true in the Julian calendar; in the Gregorian calendar it is true as long as we stay within one century, which is always the case with d